On the Diffraction of Light incident at nearly the Critical Angle on the Boundary between two Media.

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[Plate 9.]

1. *Introduction.*

When a parallel pencil of monochromatic light falls at nearly the critical angle on the plane boundary between two media of differing refractive index, and emerges in the rarer medium in a direction almost parallel to the surface of separation, the aperture of the emergent pencil becomes greatly restricted. A study of the diffraction phenomena observed in this limiting case of oblique emergence seems of interest, especially in view of the fact that in the Lummer-Gehrcke interference spectroscope we have an actual case of importance in which such obliquely emergent pencils determined the observed phenomena. Some preliminary work on this subject was carried out by Prof. C. V. Raman, and at his suggestion the author of the present paper undertook a detailed experimental investigation of the case with a view to establishing its principal features. So far as the writer is aware, though the theory of the parallel-plate interferometer in its essentials is well known,* the special character of the diffraction phenomena observed with pencils very obliquely refracted at a plane surface had not been discussed previously. It is proposed in the present paper to describe the results obtained by the writer, some of the observations (to be mentioned below) having special reference to the theory of the Lummer-Gehrcke plate.

2. *Experimental Methods and Results.*

The general features of the case may be observed on an ordinary spectrometer. It is well known that an ordinary prismatic spectroscope may be so adjusted as to give very large dispersions, by putting the prism on the table of the instrument in such a position that the light incident on the first face, after passing through it, falls at nearly the critical angle of incidence on the second face and emerges nearly grazing the surface. The lines in the spectrum then appear very widely separated, and also strongly curved, owing to the fact that pencils of light from different points on the slit of the collimator do not all pass through the principal plane of the prism. In his investigation of the optical power of spectroscopes, Lord Rayleigh* has remarked that the resolving power of the prism in the position referred to above is no greater than at other positions. The width of the beam emerging from the prism tends to zero as the grazing position is approached, and the spectrum lines are much widened by diffraction. This sets off the effect of the increased dispersion on the resolving power. Owing to the great dispersion and the enfeeblement of the light in oblique transmission, a very powerful source of monochromatic light is necessary for satisfactory observation of the diffraction phenomena. A Westinghouse quartz silica lamp is most suitable, though visual observations may also easily be made with a glass Cooper-Hewitt mercury-vapour lamp. Observations are made on the green line of the spectrum. In order to secure an extended diffraction-pattern, it is advantageous to use something less than the full aperture of the prism to start with, cutting it down by one or more slits having their edges parallel to the slit of the collimator. The position of the slits is a matter of indifference, so long as the aperture of the beam is cut down before it emerges or just as it emerges from the second surface of the prism, and not *after* it has emerged. The simplest arrangement is to limit the aperture of the beam by a slit or slits immediately before entry into the prism. Another arrangement would be to use a hollow prism, filled with liquid, and to put the slit or slits inside the prism, so as to limit the aperture of the beam just before incidence on the second face. A third arrangement is to use a glass prism the aperture of the second face of which has been reduced by grinding down to the desired width with parallel edges. All the arrangements give identical results.

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* Lummer and Gehrcke, 'Ann. d. Physik,' vol. 10, p. 457 (1903); also 'Phys. Techn. Reichsanstalt, Wiss. Abh.,' vol. 4, pp. 63–84 (1904); Kolacek, 'Ann. d. Physik,' vol. 39 pp. 997–1041 (1912); Gehrcke, 'Anwendung der Interferenzen,' Braunschweig, 1916.

Several photographs of the diffraction-pattern seen in the observing telescope were secured, of which a selection is reproduced as figs. 1, 2, and 3 in the plate. Fig. 1 represents the effect observed with a single slit limiting the aperture of the beam, the prism being in a position very nearly that of grazing emergence. It will be noticed that the diffraction-pattern is quite unlike the symmetrical system of bands usually obtained on the spectrometer with a single slit. Apart from the curvature of the bands (which has already been explained above), the pattern is markedly asymmetrical in character. To the left of the central bright fringe we have a regular succession of bands slowly decreasing in width and brightness, while on the right we have only one band, which is broad and very faint compared with the first band on the left. In general, it is found that the fringes to the right are fewer in number, broader, and much fainter than the corresponding fringes on the left. With the prism very near the critical position, the fringes to the right may even entirely vanish.

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* Rayleigh, 'Scientific Papers,' vol. 1, p. 426.

Figs. 2 and 3 represent the effects observed when the aperture of the prism is restricted by a row of equidistant parallel slits, forming a grating, the prism being nearer the critical position for fig. 3 than for fig. 2. It will be noticed that the asymmetry is very marked in these cases as well, the difference in the intensity of diffracted images of the slit to the right and to the left of the central group being specially evident in fig. 3.

3. *Photometric Study of the Diffraction Pattern.*

To put the foregoing observations on a quantitative basis, a set of photometric comparisons of the intensities of the corresponding bands of the same order on the two sides of the pattern was carried out. For this purpose, a rotating sector photometer, of a modified Abney type, was used. It was supplied by Messrs. Adam Hilger, the special feature in it being that the free disc (which can be adjusted by a handle while in rotation) is smaller in radius than the fixed disc. The sectors, while in rotation, thus present two annuli of different intensities, the ratio of which can be adjusted at pleasure by moving the handle of the instrument. The disc of the photometer is placed at the focal plane of the objective of the observing telescope, so that the diffraction pattern can be seen through it with an eyepiece, the fringes on the brighter side being observed through the inner annulus of the disc, and those on the fainter side through the outer annulus. To enable the intensities at corresponding points on the two sides of the pattern to be compared, a screen with two vertical slits is interposed immediately in front of the photometric disc, so as to cut off everything except the regions under observation, which are then adjusted to equality of brightness by moving the handle of the rotating apparatus. Several readings can be taken in succession and their average found. The angles of diffraction, θ and θ' for the two bands under comparison, can be found by reading their positions, and the position of the second surface of the prism on the divided circle of the spectrometer.

The results obtained with a single slit, limiting the aperture of the beam, are shown in column 3 of Table I. Column 1 shows the serial number of the experiment, the position of the prism on the table of the spectrometer in the successive experiments being altered so as gradually to decrease the obliquity of the emergent pencil of light.

Table I.

1.	2.	3.	4.	5.
Serial No. of experiment.	Serial order of the maxima compared.	Observed ratio of intensities.	$\mathrm{Cos}^2\theta/\cos^2\theta'$.	No. of maxima visible to the right.
1	First	4·74	4·95	1
2	First	2·55	2·52	2
2	Second	4·74	5·15	2
3	First	1·96	1·65	3
3	Second	2·56	2·54	3
3	Third	3·46	3·87	3
4	First	1·84	1·61	3
4	Second	2·30	2·18	3
4	Third	3·10	3·20	3
5	First	1·41	1·36	3
5	Second	1·84	1·74	3
5	Third	2·43	2·24	3

A few determinations were also made with a wire-grating, limiting the aperture of the beam entering the prism. The results are shown in Table II.

Table II.

1.	2.	3.	4.
Serial No. of experiment.	Serial order of maxima compared.	Observed ratio of intensities.	$\mathrm{Cos}^2\theta/\cos^2\theta'$.
1	First	5·29	5·55
2	First	3·21	3·12
3	First	2·81	3·02

In these cases, the distances between the diffracted images of the higher orders on the two sides of the pattern were too great to enable them to be seen simultaneously in the eyepiece. Consequently, the photometric observations were not extended to higher orders than the first.

4. *Apparent Failure of Snell's Law near Grazing Emergence.*

An interesting consequence of the markedly asymmetrical distribution of intensity in the diffraction pattern is that the position of maximum intensity in it is not given by the ordinary law of refraction, but is appreciably displaced to one side, so that the angle of diffraction for this position is less than the angle of refraction as given by Snell's law. The magnitude of the displacement is greatest when the emergent light practically grazes the surface of the prism, and also depends on the aperture of the pencil, being greatest when the aperture is smallest, and *vice versâ*. The displacement can be measured by first observing the position of maximum intensity when the full aperture of the prism is used, and then also when the aperture is reduced to any desired extent by the interposition of a slit before the first face of the prism. The displacement of the position of maximum intensity being negligible for the full aperture, the difference of the readings gives the magnitude of the displacement when the smaller aperture is used. On account of the great width of the central band of the diffraction pattern in such cases, it is not easy to fix very precisely upon the position of maximum intensity by direct eye-observation. Nevertheless, the displacement is clearly detectable, and can be roughly measured.

Table III shows for various apertures, the angle of refraction as given by Snell's law, and (in column 3) the position of maximum intensity in the pattern as actually observed.

Table III.

1.	2.	3.	4.
Aperture.	Angle of refraction by Snell's law.	Angle of diffraction for the central maximum. (Observed.)	Angle of diffraction for the central maximum. (Calculated.)*
mm.	° ′	° ′	° ′
1·90	88 39	88 37·2	88 37
0·70	88 20·6	88 17·8	88 17
0·60	88 14	88 10·8	88 10
0·42	88 35·4	88 31·1	88 30·6
0·30	88 25	88 19·4	88 16·8

* See Section 6.

5. *Diffraction of Light in the Lummer-Gehrcke Plate.*

As is well known, in the special form of interferometer developed by Lummer and Gehrcke, light enters a plane-parallel plate, and suffers a series of internal reflections at nearly the critical angle of incidence, and, at each such incidence, part of the light emerges as a thin pencil in a direction nearly grazing the surface of the plate. It is obvious that, if only one of such pencils entered the observing telescope, we would see in the field of view an unsymmetrical diffraction pattern of the type described in the preceding sections. Actually, the effects seen would be modified by two causes: firstly, by the interference (under large difference of path) of the several pencils emerging from the plate; and, secondly, when an extended source of light is used, by the superposition of the effects of the different parts of the source. The second complication may be avoided by narrowing down the slit of the collimator from which the light enters the plate, and setting it parallel to the plane of the plate. The interference maxima seen in the field of view would then be formed entirely by the light diffracted on emergence from the plate. The unsymmetrical character of the distribution of intensity, and its similarity with the phenomena previously described, is rendered most conspicuous by somewhat reducing the aperture of the pencil immediately before its entry into the plate, so that the number of interference maxima falling between consecutive diffraction minima becomes fairly large.

Figs. 4, 5 and 6 show the effects then observed for different angles of incidence, and make evident the great difference in intensity of the corresponding interference maxima on the two sides of the central fringe. The difference can be seen even when the full aperture of the instrument is used. This is clear from figs. 7 and 8 in the Plate, which also serve to illustrate another important feature in the theory of the Lummer-Gehrcke plate. It will be noticed that each of these figures shows the interference maxima for two different angles of incidence of the light, and the positions of these maxima, formed in diffracted light, remain unaltered—though their intensities change—when the angle of incidence is varied. It is this invariability in the positions of the interference maxima that enables an extended source of light to be used in practical work with the Lummer-Gehrcke plate without resulting in loss of sharpness of the fringes. It also explains why in figs. 4, 5 and 6 the interference fringes run practically straight, while the diffraction maxima and minima are strongly curved.*

6. *Explanation of the Results.*

(*Rewritten February* 20, 1921.)

It may be remarked that diffraction-phenomena analogous to those described above are also noticed when the pencil of light from the collimator is reflected at nearly grazing incidence from the surface of a prism and is viewed through the observing telescope. Indeed, similar effects may also be obtained very simply by merely placing two razor-edges facing opposite ways, one nearly behind the other, parallel to the spectroscope slit, on the table of the instrument and viewing the light transmitted through the very oblique aperture thus formed. In the diffraction pattern obtained by any of these methods, the fringes on the left are fewer, broader and much less intense than the fringes on the right of the central maximum, in the same way as in the case at present under consideration.*

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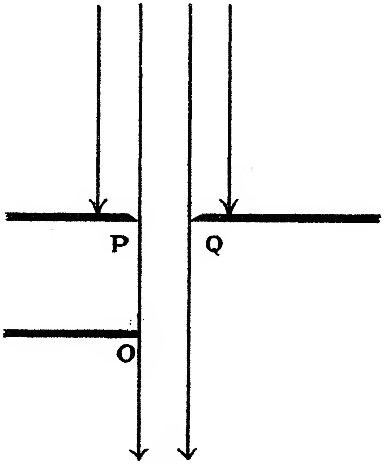
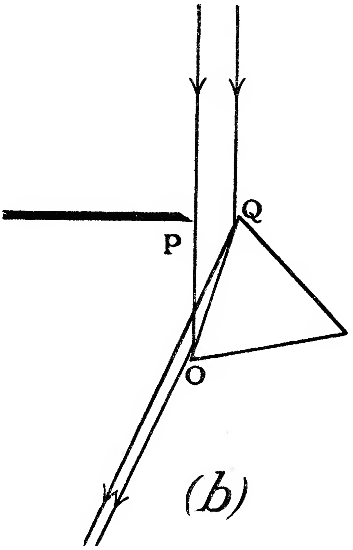
* That the positions of the interference maxima formed by the light diffracted on emergence from the plate remain unaltered when the incidence of the light at entry into the plate is varied, may be explained in a very simple manner. From the principle of minimum or stationary path (Fermat's Law), it follows that the retardation of a wave which starts at a given position and arrives at a specified point in the focal plane of the observing telescope, after one or more reflections within the plate, depends only on the number of such reflections, and is independent of any variations in its actual path, so long as such variations are small. It follows that the positions of the interference maxima are unaffected by small changes in the actual path of the light.

The referee of the Royal Society in commenting on the explanation of the phenomena put forward in this paper, has suggested two experiments which have also been tried by the author. These experiments are indicated diagrammatically in figs. (*a*) and (*b*). In the first, three razor-blades P, Q, O, were placed on the table of the spectrometer as shown, and the light transmitted through the arrangement was viewed in focus through the observing telescope.



(*a*)



(*b*)

The second experiment, fig. (*b*), consists in allowing a pencil from an aperture limited by a diaphragm to fall obliquely on a reflecting surface OQ. In both experiments, however, somewhat complex effects were noticed, which varied with the distances between the three edges or with the position of the diaphragm and the mirror and which perhaps can be best described as consisting of two or more diffraction-patterns super-imposed on each other. This is not surprising, as in figs. (*a*) or (*b*) the third edge or diaphragm introduces an additional complication which modifies the effects due to the obliquely held aperture or reflecting surface taken by itself. In any case, however, these observations did not prove to be of assistance in clearing up the nature of the effects under discussion noticed by the author.

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* The effects observed with oblique reflexion and oblique transmission have been studied by C. V. Raman, 'Phil. Mag.,' November, 1906, January, 1909, and May, 1911. See also S. K. Mitra, 'Phil. Mag.,' January, 1918.

In all the cases with which we are now concerned, the common feature is that the fully illuminated aperture from which the light finally emerges, the elements of which should be regarded as the centres of diffracted disturbance, lies in a direction nearly parallel to the direction of the diffracted rays. Dividing this aperture into equal small elements and considering the phases of the beams incident on and diffracted from these elements and summing their effects, we find that in directions given by the formula

$$a(\mu \sin i - \sin \theta) = \pm n\lambda, \qquad (1)$$

the illumination in the diffraction-pattern should be zero, i being the angle of incidence, θ the angle of diffraction, a the aperture and μ the refractive index. The intensity at any point in the diffraction-pattern as obtained by summation of the effects of the elements of the aperture is given by

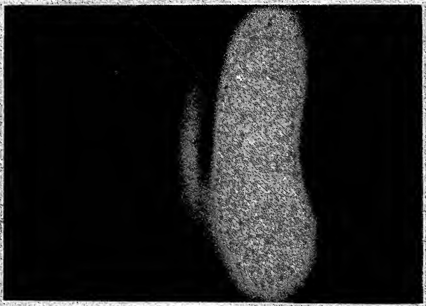
$$I = I_0 \sin^2 \phi / \phi^2, \qquad (2)$$

where

$$\phi = \frac{\pi a}{\lambda}(\mu \sin i - \sin \theta).$$

In agreement with these formulæ, we find that the dark bands in the diffraction-pattern are sensibly of zero illumination and their positions agree with theory. In the cases under consideration θ is nearly equal to $\pi/2$, and it is readily shown that the angular separation of the dark bands should increase continually from one side of the pattern to the other, exactly as observed. It is also clear why there are fewer bands on one side, as the value $\theta = \pi/2$ sets a limit to the extension of the pattern in that direction.

The feature requiring special explanation is the asymmetry in the *intensity* of the bright bands on the two sides of the pattern. If in formula (2) the quantity I_0 were a constant with respect to θ, the distribution of intensity in the pattern would be symmetrical, that is, corresponding bands on either side of it would be of equal intensity. Actually, however, this is not the case, as we have seen, and we are led to regard the observed effects as indicating a variation of the quantity I_0, with the angle of diffraction θ. Such a variation might have been expected, *a priori*, as the direction of the diffracted rays is nearly parallel to the aperture, and hence the cross-section $a \cos \theta$ of the diffracted beam varies rapidly with θ (which is nearly equal to $\pi/2$), instead of being practically constant as in the case of diffraction by a normally held aperture. The quantity I_0 is in fact proportional to the square of the amplitude of the secondary disturbance contributed by each element of the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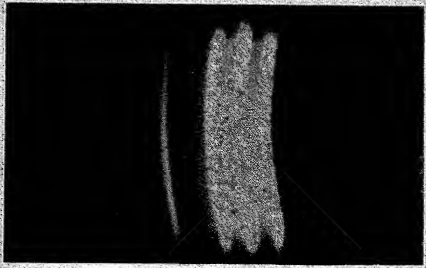
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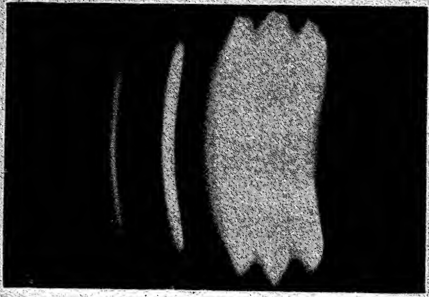
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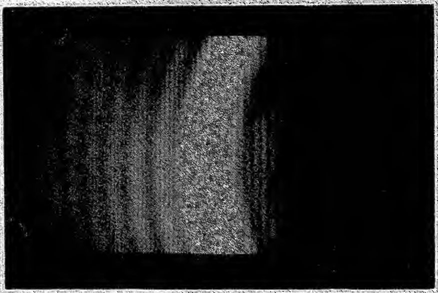
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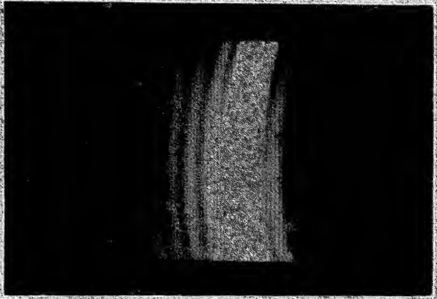
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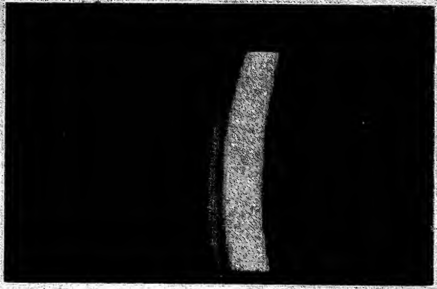
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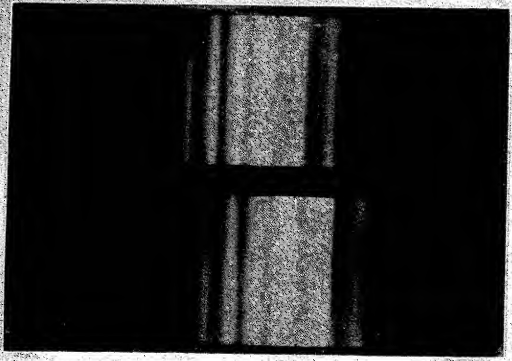
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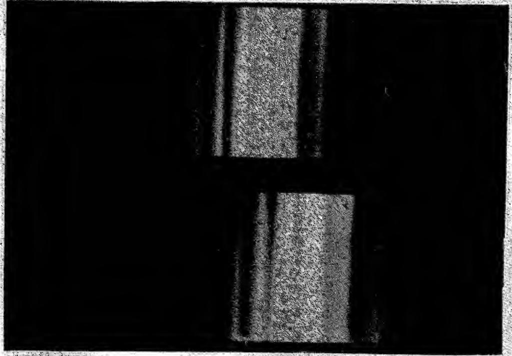
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aperture, and if we assume this amplitude to be proportional to the cosine of the angle of diffraction θ, we obtain finally

$$I = I_0 \sin^2 \phi / \phi^2 = I_1 \cos^2 \theta \sin^2 \phi / \phi^2,$$

where I_1 is sensibly a constant within the diffraction-pattern. On this basis, the ratio of the intensity of corresponding bands on either side of the pattern should be $\cos^2\theta_1 / \cos^2\theta_2$ where θ_1 and θ_2 are the respective angles of diffraction, and this, as we have seen agrees with the results of the photometric determination (Section 3, above).

The foregoing also enables us to calculate the displacement of the position of maximum intensity in the pattern from that given by Snell's law. We may obtain the position of maximum intensity from the expression for I as given above, by putting $dI/d\theta$ equal to zero. It is found that for this position, the phase-angle ϕ is not zero, but is approximately given by

$$\phi = \frac{\lambda}{\pi a} \cdot \frac{3 \sin \theta}{\cos^2 \theta}.$$

The angles of diffraction for the position of maximum intensity in the pattern have thus been calculated and shown in column 4 of Table III for comparison with experiment.

In conclusion, the author wishes to express his cordial thanks to Prof. C. V. Raman, who suggested the investigation and throughout took much interest in its progress, besides providing the necessary facilities for carrying it out at the Palit Laboratory of Physics.

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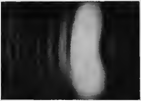

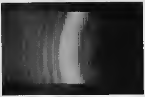

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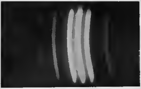

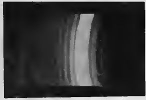

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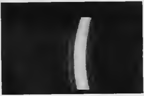

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